Exam EQP Part 4 - 13 January 2017

1 Diffusion of light in a scattering medium.

The transport of waves inside a multiple scattering medium can often be described by a random walk. The typical length scale is given by the transport mean free path ℓ, which is the length scale over which the wave loses all information on its original direction. The random walk results in a diffusion process with diffusion constant $D = \frac{1}{3}c\ell$, where c is the wave velocity.

1.1a *(2 points)* A cloud consists of finely dispersed water droplets, each of which has a scattering cross section of $\sigma = 100\ \mu\text{m}^2$. The density of droplets in the cloud is $n = 100\ \text{cm}^{-3}$. Calculate the mean free path and the diffusion constant for light propagating inside the cloud.

1.1b *(4 points)* Calculate the average diffusion time of light that is multiple scattered through this cloud, which has a thickness of 500 meters. Compare this time to the ballistic travel time, i.e. the time it takes a coherent beam of light to pass through the cloud without any scattering. An interesting possibility of communicating / imaging through clouds could be time-gating of optical signals. Discuss what kind of experiment you could do to set this up. Discuss the main limitations of this method.

1.1c *(4 points)* You are travelling with a colleague in an airplane above the cloud and notice that around the shadow of the plane on the cloud there is a bright halo of increased intensity (see Figure 1 at the end of this exam form). Your colleague suggests that this could be a coherent backscattering (cbs) effect from the cloud. Explain using your knowledge of the physics of cbs whether or not this halo can be caused by coherent backscattering from the cloud.

1.1d *(4 points)* It has been observed that the moon shows a rapid increase in brightness near opposition (earth positioned exactly between the sun and the moon). Figure 2 shows experimental data (Figure 8 from Kaydash et al.) of the lunar reflectivity. Discuss whether coherent backscattering could play a role in this case, and what would be the range and magnitude of this contribution. The lunar soil consists of a fine dust of about 10 μm particle size. Use the data provided in Figure 3 by Hapke et al, Science 1993, on reflectance measurements of Apollo lunar soil samples.

natural talent

[illegible]

1.1c *(3 points)* An important challenge in medicine is the need for capabilities to see inside the human body using noninvasive methods. A research team at Northwestern University in the USA have developed a technique for medical diagnostics of diseases relying on coherent backscattering of light. Their method relies on measuring the coherent backscattering cone while scanning a broadband illumination source over the body. Given an experimental angular resolution of 0.01°, calculate the typical depth range they can measure inside the body using this technique.

2 Coherent backscattering of light

2.1 Coherent backscattering of light in the time domain: general argument

In this question you will discuss the coherent backscattering effect and its dependence on the scattering angle and on time.

A plane wave is incident onto the medium with wavevector $\mathbf{k}_1$, entering the medium at position $(\mathbf{r}_1, t_1)$. Following diffusive transport, it exits in reflection at $(\boldsymbol{r}_2, t_2)$ with wavevector $\boldsymbol{k}_2$.

2.1a *(2 points)* Draw the corresponding scattering diagram for the diffuse far-field intensity (two-field propagator).

2.1b *(2 points)* Draw the corresponding scattering diagram responsible for coherent backscattering contribution to the intensity.

2.1c *(4 points)* Write down a general expression for the far-field backscattering intensity, denoted as α, of both the diffuse (α_d) and coherent backscattering (α_c) components in terms of an integral over the source, propagator and outgoing Green's functions. Assume that the propagator $P(\boldsymbol{\rho}_\perp, z_1, z_2, t)$ only depends on the relative transverse coordinate $\boldsymbol{\rho}_\perp = \boldsymbol{\rho}_{\perp,2} - \boldsymbol{\rho}_{\perp,1}$ and $t = t_2 - t_1$. Show that, compared to the diffuse intensity, the coherent backscattering gains an extra phase factor which cancels to zero in the exact backscattering direction ($\boldsymbol{k}_2 = -\boldsymbol{k}_1$).

2.1d *(4 points)* In the following, we approximate the extra phase factor by its transverse component, which is the part depending on $\boldsymbol{k}_\perp$ and $\boldsymbol{\rho}_\perp$. This amounts to ignoring the dependence of the phase term in z. Show that the propagation equation, which is a convolution integral in real space, becomes a product in momentum space for the transverse component.

2.2 Coherent backscattering of light in the time domain: calculation

Given that the propagation is the direct product in momentum space, it is convenient to write the time-dependent propagator for the diffuse intensity (diffuson) as the Fourier-transform to the transverse coordinate. The resulting expression for $P(k_\perp, z_1, z_2, t)$ is given by

$$P(k_\perp, z_1, z_2, t) = \frac{e^{-Dk_\perp^2 t}}{(4\pi Dt)^{1/2}} \left[\exp(-\frac{z_-^2}{4Dt}) - \exp(-\frac{z_+^2}{4Dt}) \right] \qquad (1)$$

with $z_- = z_1 - z_2$ and $z_+ = z_1 + z_2 + 2z_0$. Here z_0 is the extrapolation length, taking into account that the flux at the surface is nonzero due to leakage of light out of the medium.

2.2a *(3 points)* Show that the diffuse intensity α_d is determined only by the zero-momentum part of the propagator, $P(k_\perp = 0, z_1, z_2, t)$. Show that $\alpha_c = e^{-Dk_\perp^2 t}\alpha_d$ where $k_\perp = k_0 \sin(\theta)$ is the angle-dependent transverse wavevector between incident and scattered waves.

2.2b *(4 points)* Show that, for large times t compared to the mean free time $\tau_e = \ell/c$, the z-dependence (terms in the brackets) forms only a small correction of order $(z_0 + \ell)^2/4Dt$. Hint: assume the penetration depth of the light is of the order of one mean free path ℓ.

2.2c *(4 points)* Estimate the time dependence of the diffuse intensity reflected from a semi-infinite slab and draw the time-dependent path length distribution on a log scale.

2.2d *(4 points)* Write α_c as a function of the backscattering angle θ. Sketch qualitatively the time-dependence of the CBS intensity $\alpha_c(\theta)$ for selected angles including exactly zero, an angle in the flank of the cbs cone, and a large angle (outside the cone).

2.2e *(3 points)* Sketch the shape of the coherent backscattering peak for different times t corresponding to 1, 2, 3, etc mean free times. Qualitatively, describe how the far-field angular distribution relates to the distance between first and last scatterer in the medium.

2.2f *(3 points)* In an article by Muskens et al., Phys. Rev. B 2011, the angle dependence of frequency correlations in the speckle was measured in the cbs cone. Use the known relationship between the time-response and frequency correlation to estimate qualitatively how the frequency correlation width in the coherent backscattering cone depends on angle.

2.2g *(3 points)* The total intensity is given by a time integral of the time-dependent intensity, i.e. $\alpha_{c,\text{tot}} = \int dt\, \alpha_c(t)$. In the small-angle approximation, $k_\perp \simeq k|\theta|$, show that the total coherent intensity is given by

$$\alpha_c(\theta) = \int_{\tau_e}^{\infty} t^{-3/2} e^{-t/\tau_\gamma(\theta)} dt \tag{2}$$

This expression allows to define an angle-dependent coherence time, $\tau_\gamma(\theta)$, which defines the path lengths contributing to a certain angular range. Define τ_γ in terms of θ and the mean free path ℓ.

2.2h *(2 points)* Explain how absorption can be take into account though τ_γ. Draw the shape of a coherent backscattering cone, for (1) a material without absorption and (2) a material with absorption. Assume an absorption length of several transport mean free paths.

2.2i *(2 points)* In an article published by Wiersma et al., Phys. Rev. Lett. (1996), coherent backscattering cones were reported in an amplifying medium. Here, optical gain was provided by stimulated emission of radiation using a laser dye infiltrated into a suspension of white TiO_2 paint. Assuming exponential increase of the intensity in presence of gain, qualitative sketch and explain in words the shape of a cbs cone for the case of a gain length equal to several transport mean free paths.

3 Intensity fluctuations

An important aspect of random multiple scattering of waves are the fluctuations in the observables such as intensity. For completely uncorrelated disorder, all phases of paths through the medium are different, resulting in some universal statistics.

3.1a *(2 points)* Explain and draw the lowest-order scattering diagram contributing to the intensity in transmission through a multiple scattering medium.

3.1b *(3 points)* We consider a total field (or wavefunction) E which is given by the sum of a large number of pathways j, $E = \sum_j a_j$, with $a_j = |a_j|\exp(i\phi_j)$ the contribution of individual pathways. Show that the average field $\langle E\rangle = 0$. Show that the variance $\Delta I = \langle I^2\rangle - \langle I\rangle^2$ of the intensity (or probability) $I = EE^*$, is equal to the square of the average intensity.

3.1c *(4 points)* By counting the combinations of lowest order scattering diagrams, derive that $\langle I^n \rangle = n! \langle I \rangle^n$.

3.1d *(4 points)* In the presence of correlated paths, the assumption of the lowest order scattering diagram is no longer valid. Describe and draw the family of scattering diagrams that contributes to the n-th moment of intensity for $n = 2$ and $n = 3$.

3.1e *(4 points)* Show that these additional contributions give rise to larger fluctuations in the intensity. The contribution of these extra correlations is proportional to the crossing probability given by the inverse of the conductance parameter, $1/g$. Describe the behavior of the intensity distribution in the limit $g \simeq 1$.

4 Anderson localization

During the course we have discussed a number of results and articles in which experiments are presented aimed at demonstrating localization of classical and quantum waves.

4.1a *(4 points)* Give an overview of the types of experiments that are done to identify Anderson localization.

4.1b *(4 points)* Select two of your favorite experimental methods and describe into more detail what is measured, how it is measured, and how data is interpreted.

4.1c *(4 points)* Critically discuss some of the most important pitfalls in the search for localization effects.

4.1d *(4 points)* Discuss the Ioffe Regel criterium $k\ell \simeq 1$ and the Thouless criterium $g = 1$ as indicators for localization. Describe the essentials of scaling theory.

Figure 1: Opposition effect of an airplane on a cloud.

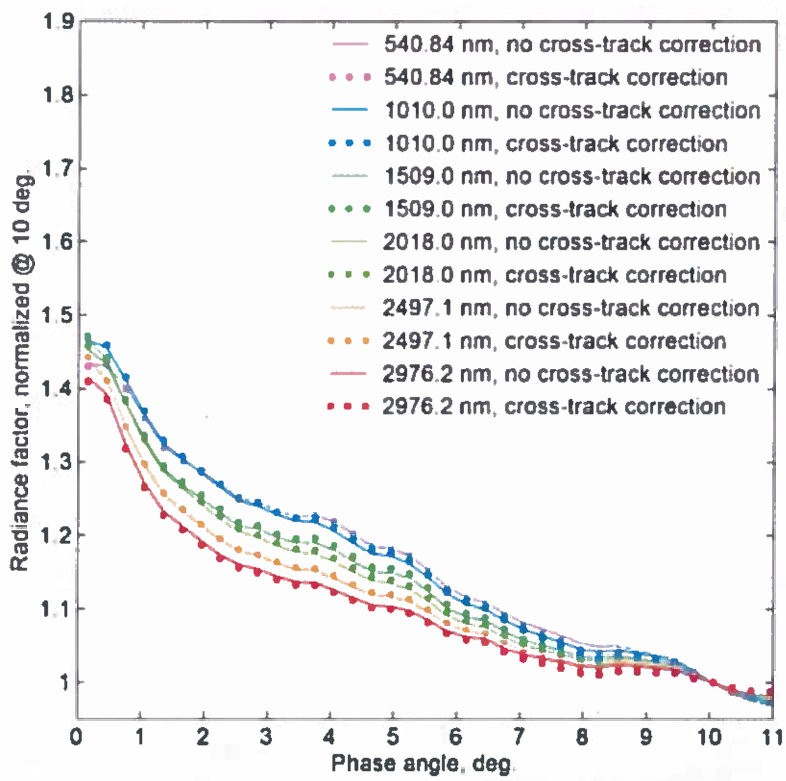

Figure 8. Phase functions for selected spectral bands. Each curve is normalized on its own value at $\alpha = 10°$. Dots show phase curves derived from M^3 data subjected to the cross-track correction [*Besse et al.*, 2013], solid curves present phase curves derived from noncorrected M^3 data.

Figure 2: from Kaydash et al., "Lunar opposition effect as inferred from Chandrayaan-1 M3 data", J. Geoph. Res. 2013. Figure 8 : radiance factor of lunar reflectance close to opposition angle. Phase angle in astronomy means the angle between the sun, the moon and the earth, zero angle indicating exact opposition (earth between moon and sun).

Table 1. Lunar samples and their normal albedo.

Sample number	Normal albedo	
	Blue (λ = 442 nm)	Red (λ = 633 nm)
10084	0.058	0.077
15041	0.065	0.094
15271	0.093	0.128
15601	0.076	0.102
61221	0.323	0.375
65701	0.116	0.153
75121	0.064	0.083
79221	0.063	0.084

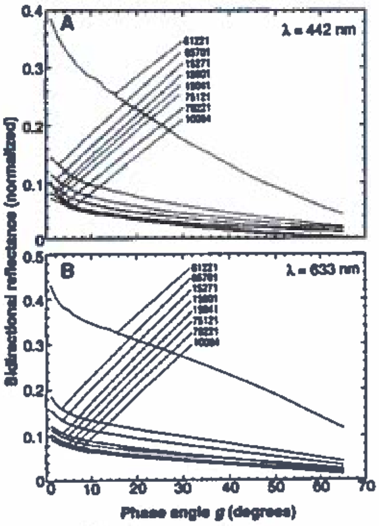

Fig. 1. Bidirectional reflectances of the lunar samples as a function of phase angle g in (**A**) blue light and (**B**) red light, normalized to their normal albedos, which are brightnesses relative to a halon standard at an incidence angle of 5°, viewing angle of 10°, and phase angle of 5° (Table 1).

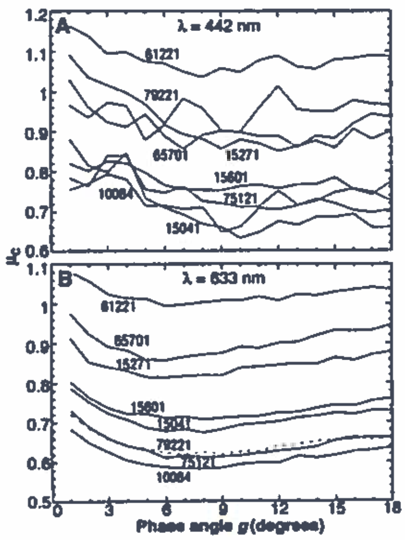

Fig. 3. Circular polarization ratio μ_C versus phase angle g in (**A**) blue light and (**B**) red light. The helicity of the incident irradiance is left-handed (*21*).

Figure 3: From Hapke et al., Science 1993. Table 1: albedo (reflective efficiency) of different Apollo samples of lunar soil. Fig. 1: Far-field scattering intensity versus detection angle. Fig. 3: Far-field intensity polarization ratio, i.e. parallel polarization normalized to the cross-polarization intensity. Parallel means that incident and detected polarizations are the same, crossed means that the polarizations are perpendicular. Circular polarizations were used.