Structure of Matter (NS-266B)

Test 3 – Nuclear Physics

3 April 2014; time: 45 minutes

The test consists of two parts (I and II):
- In Part I, basic knowledge is tested in ten multiple-choice questions.
- Part II consists of two open exercises.

Each multiple-choice question gives 1 point. For the open exercise the maximal number of points is 10 for each exercise. **The total number of points is 30.**

→ Answer the multiple-choice questions on one piece of paper.
→ Answer each of the open exercises on a separate piece of paper.
→ Write your name and student number on each page.
→ Do not give final answers only, explain your reasoning (short) and/or give full calculations.
→ Calculator use is allowed (no cell/smart phone!).

Success!

Part I: Multiple choice questions

Instructions: Choose one answer. Each correct answer gives 1 point.

1. The momentum of an elementary particle is typically given in not-natural units, namely

   A. GeV
   B. GeV/c
   C. GeV·c
   D. GeV/c^2

2. The Rutherford's scattering law is true when the α-particle

   A. is absorbed by the gold nucleus.
   B. penetrates the nucleus.
   C. undergoes a deep inelastic scattering.
   D. has a head-on collision on the gold nucleus.

3. The radius of a nucleus is how many times smaller than the one of an atom?

   A. 100
   B. 1'000
   C. 100'000
   D. 10'000'000

4. The sub-structure of the proton and neutron is evident from its

   A. mass.
   B. electric charge.
   C. colour charge.
   D. magnetic moment.

5. The relevant interaction for the decay $n \rightarrow p + e^- + \overline{\nu}_e$ is the

   A. strong interaction.
   B. gravitational interaction.
   C. weak interaction.
   D. electrostatic interaction.

6. An indication for the existence of a neutral, almost mass-less particle (called neutrino) comes from the

   A. fast decay of certain nuclei species.
   B. continuous spectrum of the electron in the β decay.
   C. discrete spectrum of the electron in the β decay.
   D. excitation spectrum of β-particle emitting nuclei.

7. At current energies in the universe the nuclear force is

   A. is a fundamental force.
   B. as strong as the electro-magnetic force.
   C. residual of the electro-magnetic force.
   D. residual of the strong force.

8. The nuclear interaction between two nucleons can be considered as the exchange of

   A. mesons.
   B. hyperons.
   C. leptons.
   D. photons.

9. The deviations from the Bethe-Weizsäcker mass formula are observed for nuclei with

   A. no mass defect.
   B. magic decay length.
   C. extremely high binding energies.
   D. extremely low binding energies.

10. The basic idea of the nuclear shell model is that

   A. the nucleus has a shell structure on which the nucleons are fixed.
   B. in first approximation each nucleon moves independently in a common potential field.
   C. the spin-orbit interaction can be neglected.
   D. the nuclear potential has a sombrero shape.

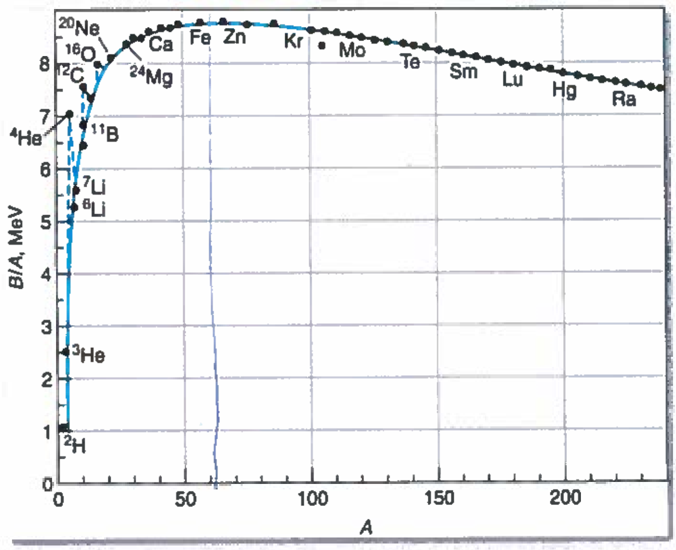
Part II: Open questions

Exercise 1: Bethe-Weizsäcker mass formula (10 points)

a) (2 points) What is the atomic mass A and the atomic number Z for the following isotopes?

$$^{240}_{94}Pu_{146} \qquad \text{and} \qquad ^{60}_{28}Ni_{32}$$

b) (2 points) Determine the binding energy E_B/A of both isotopes using the figure below?



c) (6 points) We consider the mass formula in a reduced form (volume, surface and Coulomb term)

$$E_B = a_V \cdot A - a_S \cdot A^{2/3} - \frac{3}{5}\frac{Z^2 e^2}{4\pi\varepsilon_0 \cdot r_0 \cdot A^{1/3}}$$

Determine the constants a_v and a_s.

Useful parameters:

$r_0 = 1{,}2\,fm$

$$\frac{e^2}{4\pi\varepsilon_0} = 1{,}44\,MeV \cdot fm$$

Exercise 2: Radioactive decays (10 points)

In nature we can find 3 types of Uranium isotopes:

- ${}^{238}_{92}U$, which forms the 99.27 % of the Uranium in nature and decays with a $t_{1/2} = 4.47 \cdot 10^9$ years
- ${}^{235}_{92}U$, which forms the 0.72 % of the Uranium in nature and decays with a $t_{1/2} = 7.04 \cdot 10^8$ years
- ${}^{234}_{92}U$, which forms the 0.01 % of the Uranium in nature and decays with a $t_{1/2} = 2.45 \cdot 10^5$ years

a) (1 point) Explain why ${}^{234}_{92}U$ is the less abundant isotope (short answer)

b) (4 points) Consider the two most abundant isotopes (${}^{238}_{92}U$ and ${}^{235}_{92}U$). Under the realistic hypothesis that when the solar system was created their abundance was the same, calculate the age of the solar system

c) (2 points) Consider now a decay chain like

$$A(\lambda_a) \to B(\lambda_b) \to C$$

where λ_a is the decay constant for the nucleus A, and λ_b is the is the decay constant for the nucleus B. Write the differential decay equation for the intermediate stage B (the equation from which you can calculate the number of nuclei $N_B(t)$)

d) (3 points) Consider know the decay chain of ${}^{234}_{92}U$

$${}^{234}_{92}U \to {}^{230}_{90}Th \to {}^{226}_{88}Ra$$

Knowing that the solution to the differential equation of c) is

$$N_B(t) = \frac{\lambda_a N_0^A}{\lambda_b - \lambda_a}\left(e^{-\lambda_a t} - e^{-\lambda_b t}\right)$$

and that the half-life time for ${}^{230}_{90}Th$ is $t_{1/2} = 75000$ y, find at what time you will have the maximum amount of ${}^{230}_{90}Th$