Midterm Cosmology (ns-tp430m)

Apr 15 2008 2-5pm Rm 206 Minnaert (20 points, 30%)

1. Schwarzschild - de Sitter metric. (*5 points*)

The Schwarzschild - de Sitter space-time metric is of the form,

$$ds^2 = \left(1 - \frac{2G_N M}{c^2 r} - \frac{\Lambda r^2}{3c^2}\right) c^2 dt^2 - \left(1 - \frac{2G_N M}{c^2 r} - \frac{\Lambda r^2}{3}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2(\theta) d\phi^2\right) . \qquad (1)$$

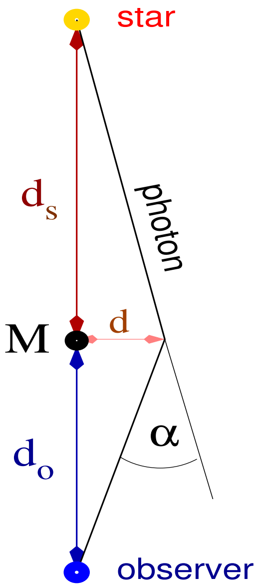

FIG. 1: The geometry of light deflection for Problem 1.a.

(a) *Light deflection.* (*3 points*)

Consider the geometry in which the source (a star) and the observer (on Earth) are symmetrically placed with respect to the lense of mass M as shown in figure. Calculate the light deflection angle α as a function of the impact parameter d (the closest distance of the photons to the lense of mass M at $r = 0$). Express your answer as a function of the distance of the observer to the center of mass d_o and of the distance of the source to the center of mass d_s. For simplicity assume a small deflection angle $\alpha \ll 1$, $d_s \gg d$ and $d_o \gg d$. What is the distance $d_o + d_s$ to the star at which the deflection angle vanishes?

The following integral you may find useful,

$$\int \frac{dy}{(x^2 + y^2)^{3/2}} = \frac{y}{x^2 \sqrt{x^2 + y^2}} . \qquad (2)$$

(b) (*2 points*)

Consider the Universe with $\Omega_\Lambda = \Lambda/(3H_0^2) = 0.74$ and $\Omega_{\rm m} \equiv \rho_m/\rho_{\rm cr} = 0.26$, $\rho_{\rm cr} = 3c^2H_0^2/(8\pi G_N)$, where $H_0 = 0.71$ km/s/Mpc ($H_0^{-1} \simeq 14$ Gy) is the current Hubble parameter. What is the Hubble parameter and the Hubble radius in the limit when $t \to \infty$? (Based on the article: *"The End of Cosmology,"* Scientific American, Mar 2008.)

2. Flatness problem. (*3 points*)

The measured value of the curvature density parameter $\Omega_k(t_0) = -c^2k/(a^2H^2)_0$ is today $-0.0175 < \Omega_k < 0.0085$ (95%CL: WMAP 2008). Calculate the dependence of $\Omega_{\rm k} = \Omega_{\rm k}(a)$ on the scale factor a in radiation era and in inflation driven by a scalar potential with the equation of state, $\mathcal{P}_\phi = w_\phi \rho_\phi$, where $|w_\phi + 1| \ll 1$ and $w_\phi =$ constant. State the flatness problem! How is it resolved by an inflationary epoch?

3. Particle horizon. (*2 points*)

Calculate the physical ($\ell_{\rm ph}$) and comoving particle horizon ($\ell_{\rm c}$) in de Sitter inflation. Express your answers in terms of the Hubble radius, $R_H = c/H_I$, where H_I is the inflationary Hubble parameter. Based on your calculation can you conclude that two particles which are at a super-Hubble distance at a time t can come in causal contact at some later time $t' > t$? Justify your answer! *Hint:* Recall that the comoving particle horizon is defined by, $\ell_{\rm c}(\eta) = c(\eta - \eta_{\rm in})$, where $\eta_{\rm in}$ and η denote the initial and current conformal time.

4. Confomal diagram. (*5 points*)

(a) (*3 points*) Draw the conformal diagram (and write the corresponding line element in terms of conformal time η and proper radial distance χ) for a closed radiation dominated universe, with $\Omega_{\rm r} + \Omega_{\rm k} = 1$, where $\Omega_{\rm r} = \rho_{\rm r}/\rho_{\rm cr}$, $\Omega_{\rm k} = -c^2k/a^2 < 0$ and $\rho_{\rm cr} = (3c^2H_0^2)/(8\pi G_N)$, H_0 is the Hubble parameter today and $k = 1/R_{\rm curv}^2$ denotes the spatial curvature of space-times. Find the scale factor $a = a(\eta)$ as a function of (conformal) time.

(b) (*2 points*)

Calculate both the physical and conformal time (after the Big Bang) at which the Universe recollapses (Big Crunch). Express your answer in terms of H_0 and $\Omega_{\rm r}$. *Hint:* The answer for the physical time at the Big Crunch is:

$$t_{\rm b.c.} = \frac{2\sqrt{\Omega_{\rm r}}}{H_0(\Omega_{\rm r} - 1)} \,. \tag{3}$$

5. Bouncing cosmology. (*5 points*)

Consider a universe with a negative spatial curvature $\kappa < 0$ whose expansion is governed by a homogeneous scalar field $\phi = \phi(t)$ with the energy density

$$\rho_\phi = -\frac{1}{2}\dot{\phi}^2 \tag{4}$$

(note the 'wrong' sign of the kinetic term).

(a) (*3 points*)

Solve the corresponding scalar equation of motion and the Friedmann equations. Find the scale factor $a = a(\eta)$ and the Hubble parameter as a function of (conformal) time.

(b) (*1 point*) Find the maximum expansion rate and the minimum scale factor.

(c) (*1 point*) What is the principal advantage of a bouncing cosmology model like this one with respect to the standard Big Bang cosmology?

Bouncing cosmologies were considered by Tolman and others in the 1930s, and they have experienced a revival in the last decade.

The following integral you may find useful:

$$\int \frac{dx}{\sqrt{x^2 - b^2}} = \text{Arccos}\left(\frac{x}{b}\right) . \tag{5}$$