Institute for Theoretical Physics, Faculty of Physics and Astronomy, UU.
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In 2004/2005, the course NS-TP401M was given by Stefan Vandoren.

Quantum Field Theory (NS-TP401M)
November 18th 2004

Question 1. Operator quantisation

Consider the Lagrangian of a free scalar field ϕ in d space-time dimensions,

$$L = -\frac{1}{2}\partial^\mu \phi \partial_\mu \phi - m^2\phi^2. \tag{1}$$

* Define the canonical momentum $\pi(\vec{x},t)$ and write down the Hamiltonian $H(\pi,\phi)$.

* Quantise the system by decomposing the field and its momentum in terms of creation and annihilation operators $a(\vec{k})$ and $a^\dagger(\vec{k})$ with comutation relations

$$\left[a(\vec{k}), a^\dagger(\vec{k})\right] = \delta^{d-1}(\vec{k}-\vec{k}'). \tag{2}$$

* Compute the commutator

$$[\pi(\vec{x},t), \pi(\vec{x}',t')] \tag{3}$$

and show that when $(x-x')^\mu$ is a spacelike vector in Minkowski space, the commutator vanishes (you may use that $\int \mathrm{d}^{d-1}k/2k_0$ is Lorentz invariant).

Question 2. Path integrals and correlation functions

The path integral, including sources $J(x)$, can be written as

$$W_J = \exp\left(\frac{i}{\hbar}S_{int}(\frac{\delta}{\delta J(x)})\right)\exp\left(\frac{1}{2}(J,\Delta J)\right), \tag{4}$$

Where S_int denotes the interaction terms, $\Delta(x-y)$ is the propagator, and we use the notation that $(J,\Delta) \equiv \int d^dx \int d^dy J(x)\Delta(x-y)J(y)$.
The Lagrangian we consider is

$$L = -\frac{1}{2}\partial^\mu \phi \partial_\mu \phi - m^2\phi^2 - g\phi^3 \tag{5}$$

* First consider the free Lagrangian, i.e. when $g = 0$ and so $S_{int} = 0$. Compute the (disconnected) four-point correlation function by taking functional derivatives of W_J with respect to the source. Draw the corresponding Feynman diagrams.

* Now switch on the interaction by taking $g \neq 0$, and expand the path integral W_J to order g^2. Compute the four-point correation function (at order g^2) at the classical level, i.e. without terms that correspond to loop diagrams.

* Draw the corresponding Feynman diagrams and explain the combinatorial factor.