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Electrodynamics
Midterm Exam

Date: *Friday, May 19th 2017*
Duration: *2 hours*
Total: *22 points*

1. *Use a separate sheet for every exercise.*
2. *Write your name and initials on all sheets, on the first sheet also your address and your student ID number.*
3. *Write clearly, unreadable work cannot be corrected.*
4. *You may use the book of Griffiths.*
5. *Distribute your time wisely between the exercises.*

1. Potential of a charged sphere (5 points)

We consider a sphere of radius R centered at the origin. We assume that the sphere carries the charge density $\rho(r,\theta) = k\frac{R}{r^2}(R-2r)\sin\theta$, where k is a constant and r, θ are the usual spherical coordinates.

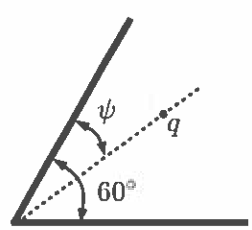
a) Derive the total charge of the charge distribution. (2 points)

b) Derive the dipole contribution of the approximate potential for points on the z-axis, far from the sphere. (2 points)

c) One can show that the quadrupole contribution to the approximate potential is non-vanishing along the z-axis. On this axis, what is the power $1/r^n$ of the leading term that dominates the behaviour of the approximate potential far from the sphere? (1 point)

2. Image charges (7 points)

Consider a point charge q located at an arbitrary point P between two grounded conducting metal plates tilted at an angle of 60° as indicated in the figure.



a) Calculate the potential $V(\mathbf{r})$ in the space between the plates using the method of images. Sketch a figure that shows the required image charges. Discuss how the result depends on the angle ψ indicated in the figure. (4 points)

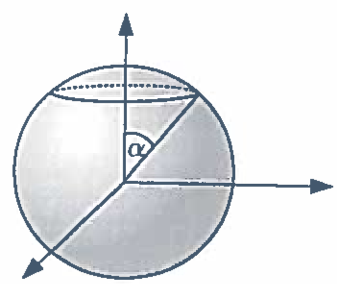
b) How much work **W** did it take to bring in the charge q from infinity? (1 points)

c) Can one use the method of images to determine the field outside of the two plates? Explain your answer. (1 point)

d) What is the relation of the angle between the plates and the number of image charges? (1 point)

3. Charged sphere with a cap (10 points)

A hollow sphere of radius R admits a specified potential $V_0 = \frac{Q}{4\pi\epsilon_0 R}$ on the surface, except for a spherical cap at the north pole, defined by the cone $\theta = \alpha$. The spherical cap is grounded to vanishing potential $V_0 = 0$, $\theta < \alpha$. Here r, θ, ϕ are the usual spherical coordinates whose origin coincides with the center of the sphere.



a) Use Rodrigues' formula for the Legendre polynomials $P_n(x)$ to show (2 points)

$$(2l+1)P_l(x) = \frac{d}{dx}\left(P_{l+1}(x) - P_{l-1}(x)\right) . \tag{1}$$

(*Hint: You may continue to the next part even if you are unable to show* (1).)

b) Use the separation of variables method, as discussed for a more general situation in the lecture, and equation (1) to show that the potential inside the sphere takes the form

$$V(r,\theta) = \frac{Q}{8\pi\epsilon_0} \sum_{l=0}^{\infty} \frac{r^l}{R^{l+1}} \left[P_{l+1}(\cos\alpha) - P_{l-1}(\cos\alpha)\right] P_l(\cos\theta) , \tag{2}$$

where we have set $P_{-1}(x) = -1$. (3 points)

c) Evaluate the above potential V for $\alpha = 0$ and interpret your answer. (2 points)

d) Derive the electric field **E** at the center of the sphere for general α. Into which Cartesian coordinate direction does the field **E** point? (3 points)