Utrecht University
Faculty of Science
Department of Information and Computing Sciences

Final Exam Simulation, Thursday April 19, 2012, 14.00-17.00 hr.

- Switch off your mobile phone, PDA and any other mobile device and put it far away.
- This exam consists of 10 questions
- Answers may be provided in either Dutch or English.
- All your answers should be clearly written down and provide a clear explanation. Unreadable or unclear answers may be judged as false.
- Please write down your name and student number on every exam paper that you hand in. Hand in this exam completely together with your answers on exam papers.
- A statistical table is attached.
- The maximum score (100 in total) is divided as follows:

Question	Score
1	10
2	10
3	15
4	5
5	5
6	10
7	15
8	10
9	10
10	10

Good luck, veel succes !

We consider a variant of the *job shop scheduling* problem. We have m machines that have to execute the jobs that arrive at the system. There are n types of jobs. Jobs arrive according to a Poisson process with intensity λ. The type of the job is random. On average, there is an equal number of jobs for each type $1, 2, .., n$.

A job of type j consists of n_j operations O_{jk} which have to be performed in a given order, where O_{jk} denotes the k-th operation of a job of type j. Each operation has to be executed by a given machine, where $m(O_{jk})$ denotes the machine on which O_{jk} has to be executed. The processing times of the operations follow an exponential distribution with average p. The machine are continuously available from time zero onwards.

We are going to develop a discrete-event simulation model for the job-shop scheduling problem. One of the goals of the simulation study is to analyze the *average length of the queue* before each of the machines.

(1) Which events are included in the event-scheduling model for this problem? Draw an event graph and indicate the time delay on each of the arcs.

(2) What are the state variables that have to be included in the simulation?

(3) Describe in words or pseudo code the event-handlers of the event(s) in your model. Include the computation of the average length of the queues before each of the machines.

(4) Give one other performance measure that is relevant for this simulation study. For this performance measure, show how it should be computed within the simulation program.

(5) Assume that the processing times of operations follow a uniform distribution on $[0.9p, 1.1p]$ instead of an exponential distribution with average p. What will be the effect on the average queue lengths? Explain your answer.

(6) Describe how to perform a decent output analysis to analyze the average queue length before each of the machines? Your description should include:

- Identification of the type of simulation with respect to output analysis.
- Experimental set up.
- Algorithm for the computation of a 95 % confidence interval. Also describe the meaning of this interval.

We simplify the arrival proces of the jobs. There is a collection of k_1 jobs of type 1, k_2 jobs of type 2, etc.... The *complete* collection of jobs is available at time zero, i.e. there is no arrival sequence but all jobs are available from the start. The processing times of the operations are *not* identically distributed. The processing time of operation O_{jk} follows an exponential distribution with average p_{jk}.

(7) Our purpose is to find a schedule that minimizes the completion time of the last operation. Formulate this problem as a combined simulation and optimization problem. Describe the decision variables, objective function and constraints. At which point do you have to perform a simulation and what is the output of this simulation?

We now consider a single machine scheduling problem. We are given one machine which is continuously available from time zero onwards. There are n jobs $j = 1, 2, \ldots, n$. Each job has a due date, denoted by d_j. The processing time of job j follows a $Gamma(p_j, 1)$ distribution. We want to find a schedule that maximizes the number of on time jobs, i.e., where a job is considered to be on time if the probability that it is completed by the due date is at least a 95 %.

(8) How can we generate the processing times in a program written in an imperative programming language like Java without using *any* specific random generation libraries or functions? **Note:** You do not have to give a program, but just a description or pseudo-code.

(9) For deterministic processing times clearly a job is on time if and only if it is completed by its due date. It is known that in the deterministic case the number of on-time jobs can be maximized by Moore-Hodgson's algorithm. Explain how this algorithm can be applied to solve the above stochastic problem.

(10) We consider a internet computer shop which delivers from stock (so this is not Dell). For a certain type of laptop, it is known that the average demand per four weeks is 40 with a variance of 36 per four weeks. The weekly demands are assumed to be independent. The shop wants to use the (r, q)-model as a basis for its inventory management.

- Explain the (r, q)-model
- Given the fixed ordering cost 100 and the holding cost 0.2 EURO per item per week, determine the optimal value of q.
- Given a constant lead time of one week and a desired Stock-Out-Probability(SOP) = 0.025, compute the value of r. What is the size of the safety stock in this example? *A statistical table is attached.*